# STEM from Space 

# Basic Physics \& Maths behind Chandrayaan 3 Mission 

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It is interesting to note how some of the fundamental Science, Technology, Engineering \& Mathematics (STEM) concepts learnt in the High School can get reinforced by looking at the recently launched Chandrayaan - $(\mathrm{CH} 3)$ for exploring the Moon. While I was interacting with the school students, they came up with a number of questions and these FAQs may be relevant for a much larger audience and hence this compilation.

Based on the requests from some of the readers. I would cover the dynamics of the lander's descent phase, whether powered or on a free fall. We will also get hopefully an opportunity to discuss chemistry concepts like hydrogen bonds etc, when we discuss the payloads and the expected data from them. In the one of the sets, I will pictorially depict how the kinetic energy removal at the right point in a hyperbolic trajectory, will lead not just the lunar capture, but also with the desired lunar orbit parameters.

I may have to add a disclaimer that if something is said wrong, the fault is only mine. But with the diversity of readers, such items can be addressed quickly enough. Please pass on these FAQ's to younger fold at home so that their STEM concepts will get strengthened and they will also understand and enjoy the CH 3 events as they unfold.

For simplicity, we will take Earth and Moon to be perfectly spherical, so that for points outside the force on any mass we can assume their full mass to be at its centre. The only force we consider is the Gravitational attraction. Mass of CH3 is negligible compared to masses of Earth and Moon.

| Parameter | Earth | Moon |
| :--- | :--- | :--- |
| "GM", called $\mu$, in $\mathrm{km}^{3} / \mathrm{s}^{2}$ | 398600 | 4900 |
| Radius (km) | 6378.137 | 1736 |
| Mean distance from <br> Earth (km) | - | 384,400 |
| Escape velocity at <br> surface $(\mathrm{km} / \mathrm{s}$ ) | 11.2 | 2.38 |

As Earth is 81.3 times heavier than Moon, Moon exerts roughly the same force (magnitude only) as Earth, if it is 9 time nearer (inverse square law). E.g., if CH3 is at $360,000 \mathrm{~km}$ from Earth, Moon at $40,000 \mathrm{~km}$ exerts comparable force. Thus, when distance Moon is far more than this $1 / 9^{\text {th }}$ factor, we can neglect theeffect of Moon and treat it as 2 body problem (Earth-CH3), When Earth is
much more than 9 times the Moon distance, it will again reduce to a 2-body problem (Moon-CH3). The region where Moon's gravity cannot be ignored is referred to as its Sphere of Influence (SOI).

FAQ1. What is the difference between an orbit and a trajectory?
Total energy (TE) is the sum of Kinetic Energy and Potential Energy. For 2-body problem, per unit mass, $T E=v^{2} / 2-\mu / r$, where $v$ is the velocity and $r$ is the distance from the central body.

If $T E<0$, i.e. total energy is negative, it is in a closed path around the central body, called its ORBIT

If TE> 0 , it is no longer bound to the central body and it moves off (i.e. escapes) in a trajectory.
FAQ2: If a stone is thrown by a man with a horizontal push, do we get a trajectory?

No. TE < O and so it would attempt to be in a orbit. But the Earth's surface blocks it.

FAQ3: How do we change the size of an orbit, keeping it in the same plane?
If we want to increase the apogee keeping the perigee same in an Earth's orbit, we give an increase in velocity at perigee (+ deltaV). If apogee is to be decreased, a decrease in perigee velocity (- deltaV) is to be given.

Note that +deltaV happens when the thruster firing direction is opposite to the velocity vector. If the spacecraft is moving from left to right, if the thruster fires with its plume going from right to left, the spacecraft speeds up ( $+\Delta v$ ). This is a good illustration of Newton's III law, arising out of the law of conservation of linear momentum.

To reduce the speed, i.e. to achieve $-\Delta v$, we do a retrofiring, i.e. the thruster firing direction is along the velocity vector and the "reaction force" on the spacecraft is from right to left, reducing the speed.

Also, note that for $+\Delta v$ and $-\Delta v$, the same thruster can be used if the spacecraft can be rotated by 180 degrees at the time of firing. Such orientation changes are achievable using the Attitude Control System.

FAQ4: Why was a EBA manoeuvre carried out at apogee on $16^{\text {th }}$ July?
This was a +deltaV manoeuvre at apogee to raise the perigee from the originally launched 173 km to about 220 km , to prevent a decrease in velocity due to atmospheric drag at low heights (which acts like-deltaV maneuvre).

FAQ5: Why were several EBN's carried out? Couldn't we have carried out a
large burn and imparted all the required + deltaV for getting to the Lunar Transfer orbit?

With liquid motors, we can start and stop as we wish. The first burn can be used to precisely calculate the actual deltaV obtained, based on pre and post orbits. These calibrations allow quite precise deltaV to be targeted in all later burns.

Moreover, if we wish to combine all the 5 perigee burns into one, the burn duration will be large, say 90 minutes which will cover a very large arc around perigee. Ideally, we wish to deliver all the deltaV at the perigee point only.

FAQ6: What are the required conditions for the choice of the TLI orbit?
The post-TLI orbit is highly eccentric, $(218 \times 369018 \mathrm{~km})$, but still has a total energy that is negative and hence is Earth-bound. Its path is so chosen that it comes into a close vicinity of Moon, so that Moon's gravitational force on CH 3 has to be taken into account to work out its path.

The rendezvous conditions can be met only if the correct 3 dimensional geometry is considered, as the Moon's orbit round the Earth is in a different plane compared to the plane in which CH 3 orbits.

FAQ7: What is the importance of the magnitude and timing of the Lunar Orbit Insertion (LOI) firing?
LBN\#1 or LOI is most critical as this transfer from an Earth-bound orbit to a Moon-bound orbit. This requires that CH 3 at the end of this burn has targets like the injection conditions of our SLV's. At the end of LOI, it should be having a horizontal velocity, whose magnitude should be less than the escape velocity for moon at that altitude. For example, at 118 km altitude to Moon, the escape velocity is $2.3 \mathrm{~km} / \mathrm{sec}$, and so we could target a velocity like $2.2 \mathrm{~km} / \mathrm{sec}$ so that we can get an elliptical orbit of $118 \times 18,500 \mathrm{~km}$ around the Moon, and bound to the Moon.

LOI burn will be-deltaV maneuver in which we will decrease the pre-burn velocity that could be higher than the escape velocity of Moon, 30 minutes of thruster firing can bring it down from $2.4 \mathrm{~km} / \mathrm{sec}$ to $2.15 \mathrm{~km} / \mathrm{sec}$

FAQ8: Why does the same thruster give higher deltaV for the same firing duration?

This is because each burn consumes propellant mass and this, in turn, decreases the mass of the remaining body in orbit.

FAQ9: Why do we capture into lunar orbit at the perilune of the new orbit?
The idea of the LOI burn is to bring down the velocity to be less than the escape velocity (or equivalently, make the total energy wrt Moon to be negative) so that it becomes Moon-bound. The post burn is an elliptical orbit around the Moon. In
this orbit it is moving the fastest at perilune (Kepler's second law) and that is what has to be achieved from a higher velocity that the pre LOI path had. At the apolune, the velocity itself is small, say . $2 \mathrm{~km} / \mathrm{sec}$ and if we reduce this value the perilune height will be negative and it would hit the Moon's surface.

FAQ10: Can we explain the concept of a circular orbit from energyconsiderations?
Yes. PE is energy due to position and so the PE has a value at every point around the central mass. Let us take a spherical Earth.

If we consider small heights compared radius if Earth we can treat Earth as flat and so draw equi-potentials as horizontal planes. At a height $h$ above the surface PE per unit mass is $g h$, where $g$ is the assumed constant acceleration due to gravity.

For satellite motion, we have to take the curved Earth, and so equipotential surfaces are concentric spheres around the centre of Earth. At an altitude h, this sphere has the radius "Radius of earth + $h$ ". If the horizontal velocity at
injection is chosen to be such that its KE is magnitude-wise half of the PE at this height, the TE is negative, implies it is in an orbit. It will be in a circular orbit and so PE does not change. As TE is conserved, KE also cannot change and the speed does not change (Only the direction of velocity changes).

# STEM from Space <br> Basic Physics \& Maths behind Chandrayaan-3 Mission <br> Set 2 

FAQ11. Is it correct to say that CH3 escaped Earth and was captured by Moon ( LOI of 5 Aug)?

No.
Moon itself is bound to Earth. Prior to the LOI burn, CH3 was in a highly elliptical orbit around Earth $218 \times 369018 \mathrm{~km}$. If Moon were not close to its path, it would have continued to orbit Earth with a period of around 230 hours.

When Moon gets too close to its path, the lunar gravity influences the path and the dynamics must be studied as a 3 body problem, $\mathrm{E}-\mathrm{M}-\mathrm{CH} 3$

FAQ12: What are the conditions for carrying out a thruster firing for lunar capture?

CH3 must be sufficiently close, have the proper orientation, in the proper orbit, and the right amount of -deltav is to be provided at the right time (i.e. the right point in its trajectory).

FAQ13: Why is a retro firing (negative deltaV) carried out for LOI?
Prior to the LOI burn, CH3 is in a hyperbolic trajectory. Unlike the ellipses, the hyperbola has an eccentricity exceeding 1, does not have a closed path and so left as it is, it would have escaped Moon. This corresponds to the case of total energy (TE) greater than 0 . In this hyperbolic trajectory, even when CH 3 goes infinitely far, it will have a positive KE and hence a positive velocity (called $\mathrm{v}_{\infty}$ ). By slowing down we reduce the KE and can bring the TE to be less than 0 to be captured by Moon, i.e bound to Moon.
By retrofiring and reducing the velocity to be less than the escape velocity (wrt Moon) at that altitude, we can make a hyperbolic arrival trajectory to an elliptical orbit around the Moon.

FAQ14: Some newspapers report that it takes 15 days for CH 3 to escape from Earth and then freely fly to Moon?

No. It never escaped Earth. See answer to FAQ11
FAQ15: What are slingshot maneuvers? Was it used in CH 3 mission?

A gravity assist, or a gravitational slingshot is a type of flyby which makes use of a third body to alter the path and/or speed of a spacecraft, to save propellant expense.

CH 3 did not use any slingshot and the orbit changes were achieved by thruster firings only.

Even LOI was achieved by a thruster firing to reduce the total energy to be less than zero so that an orbit around Moon can be achieved.

FAQ16: Was the long duration from launch to LOI due to limitations of the lift off capability of our rockets?

No.
From the injection orbit of apogee 36450 km to the Lunar Transfer orbit of apogee 369018 km , the injection velocity is only marginally higher (Instead of 10. $27 \mathrm{~km} / \mathrm{s}$ to $10.73 \mathrm{~km} / \mathrm{s}$ ) and well within our capabilities. However, for an unmanned mission the shortening the duration is not a priority. The required increase in velocity was given in instalments, so that the burn durations will cover only a small arc around the perigee and also to use the previous EBNs to calibrate the performance better.

FAQ17: In addition to the slowing down due to LOI maneuver, did CH 3 experience a large rotation of its velocity vector?

Yes. The hyperbola has two asymptotes and we characterize the angle between them as the turn angle. The lunar arrival hyperbola had an eccentricity of around 1.3 and hence a turn angle of 101 deg.

LOI burn was timed that just after the turn the velocity was brought under the escape velocity at that altitude.

## STEM from Space

## Basic Physics \& Maths behind Chandrayaan-3 Mission- Set 3

FAQ18. What is the minimum time needed to land on Moon? Can we cut down the journey time to say one day, by using more powerful rockets?

No.
The mission will consist of two major segments, namely, launch to LOI, and LOI to Landing. The first phase can be completed in about 3 days if the proper rendezvous conditions are met and the second in another 2 days.

More capable rockets can deliver more payload in each launch, but the journey time is decided by orbital geometry.

FAQ19: What decides the duration between launch and LOI?
The launch phase ends with CH 3 injection. Based on the injection position and velocity, the achieved orbit is determined. Assuming the injection is at around 200 km altitude, with the injection velocity horizontal. Then the only remaining parameter is the magnitude of the injection velocity (v).

The shortest duration for LOI would correspond to the initial orbit itself creating the conditions for lunar capture. A necessary but nor sufficient condition for rendezvous with Moon is that the initial orbit should pass through a distance from Earth centre of about 384000 km , the typical Earth-Moon distance. That means apogee should exceed 384000 km , and so the injection speed should exceed $10.74 \mathrm{~km} / \mathrm{sec}$. At this altitude, the escape velocity is 11 $\mathrm{km} / \mathrm{sec}$. If $v$ is more than $11 \mathrm{~km} / \mathrm{sec}$, it would get a hyperbolic trajectory and exit Earth.

Thus, we have a narrow range for $v(10.73$ to $11 \mathrm{~km} / \mathrm{s})$. Within this range, the larger the $v$, the period will be higher, but the time at which " $r$ " will meet the rendezvous condition happens sooner, and with less time duration of contact. E.g., in Apollo 11 mission it was about 76 hours. If we choose just around $10.74 \mathrm{~km} / \mathrm{sec}$, the period is around 230 hours and the closest approach to Moon can occur near its apogee, around 115 hours later. We can achieve LOI then within 5days, provided we satisfy several more additional conditions like the direction of approach, orientation, the precision of prediction etc.

FAQ20: What decides the period or the time taken for one revolution in the orbit? Is it the same around Moon as around Earth for the same radius?

In the $170 \times 4313 \mathrm{~km}$ orbit, the period is 6.25 hours (use Kepler's III law to derive this value when the SMA got reduced from 10855 to 3977 km).

The laws of Physics are the same. But as the mass of Moon is about $1.23 \%$ of

Earth's, the product GM which appears as $\mu$ is therefore only $4900 \mathrm{~km}^{3} / \mathrm{sec}^{2}$. Around Moon the period of a 100km altitude circular orbit will be around 118 minutes, compared 86 minutes at 100 km altitude of Earth. We will have the Propulsion module, Lander etc at the 100 km orbit and you will have occasions to benefit from this knowledge. Also, note that unlike a 100 km orbit around Earth that would decay too fast due to atmospheric drag, a 100 km orbit around Moon is quite stable.

FAQ21: As per Newton's second law the same force will produce more acceleration if the mass is less? When we have a succession of thruster firings for orbit correction, mass of the object should decrease due to the ejected mass of the fuel. Can you give us an idea of how this was seen during CH 3 mission?

In EBN\#2, for example, the thruster was fired for 926 sec and it produced a deltaV $=110.98 \mathrm{~m} / \mathrm{s}$; The same thruster was fired during LOI for 1834 sec and it produced a deltaV of $272.44 \mathrm{~m} / \mathrm{s}$. The doubling of the time gave an extra $22.5 \%$ deltaV, attributable to a $22.5 \%$ mass depletion from around 3800 to 3100 kg . From these values we can also estimate the thrust level as the rate of change of linear momentum and it works out to around 435 N

FAQ22: Given that in the EB phase thrusters were fired in the third, $9^{\text {th }}, 12^{\text {th }}$, $17^{\text {th }}$ and $20^{\text {th }}$ perigee transits, besides at Apogee 5, and if the centre times of firing are known, can you deduce the apogee and perigee of all the intermediate orbits. Assume that initial orbit is known as $170 \times 36450 \mathrm{~km}$ ?

This is a good exercise for students to get a feel of Kepler's third law that states that the square of the period is proportional to cube of the size of the orbit (specified by its semi major axis "a").

- From $170 \times 36450$, find initial $a=24,688 \mathrm{~km}$.

- Use $\mu$ of Earth to be $398600 \mathrm{~km}^{3} / \mathrm{s}^{2}$, to find Period $(T)=640$ minutes
- Since EBN\#1 was in $3^{\text {rd }}$ perigee, 2 revolutions in this orbit would take $2 \times 640=1280 \mathrm{mts}=21 \mathrm{~h} \mathrm{20m}$; If the injection was at 9.21 UT of 14 Jul ,

EBN\#1 will be centered at 6.41 UT. Indeed it was.

- From the calculated period from successive firings, divided by number of revs, we can compute the new " $a$ " and hence the new orbit size.
- For example, we note that there was firing at 9.01 UT of 25 Jul, followed by a firing at 18.43 Ut of 31 Jul . In the time lapse of $6 \mathrm{~d} 9 \mathrm{~h} 42 \mathrm{~m}, \mathrm{CH} 3 \mathrm{has}$ made 3 revs. Hence period is 3074 minutes. Applying Kepler's law wrt the previous orbit of period 1429 minutes and size $233 \times 71351 \mathrm{~km}$ (or $\mathrm{a}=$ 42176 km ), we can find new $a=70572 \mathrm{~km}$, or $236 \times 127603 \mathrm{~km}$


## Basic Physics \& Maths behind Chandrayaan-3 Mission- Set 4

You may find some of the questions in this set too elementary. Sometimes what we think everyone will know, may still require a revisit. Please pass on these FAQ's to younger fold at home so that their STEM concepts will get strengthened and they will also enjoy the CH3 events as they unfold.

FAQ23: Why is it that we can see only one side of the Moon always?
The Moon is not perfectly spherical. Gravitational forces stabilise the orientation of the egg-shaped Moon to keep the longer axis pointed always towards Earth as it orbits round the Earth. This gravity-gradient stabilisation leads to Moon rotating about its axis at the same rate of its orbital motion. This results in the same face (the near face) towards the Earth. The far side is thus never seen from the Earth.

Furthermore, Earth's surface has a lot of water. When the moon orbits round the earth, it causes tides that pull up towards the Moon. This is called tidal locking, which reinforces the above effect.

FAQ24: Why is it that the day time in Moon is about 15 days?
As the same face of the Moon is locked to the Earth, as the Moon orbits around the Earth, it makes one revolution about its axis as well. The latter results in alternating day and night for the Moon and a full cycle is about a month (around 29.5 Earth days). i.e., if you were standing on the Moon, it would take 29.5 days for Sun to rise, move up on the sky, set, and reappear on the rising horizon. Half of this, namely, around 15 days is then daytime and the other half being the night. Because the same face of Moon always is towards Earth, if you are standing on the Moon, you will see Earth in the samedirection always; but the stars and the Sun would move around in the sky.

FAQ25: What is the orbital period of the Moon?
The Moon's orbital period around the Earth is 27.3 Earth days. We can verify this by applying Kepler's III law w.r.t. any other known satellite like GEO radius of $42,000 \mathrm{~km}$, period 1 day and Moon's orbital radius of around $384,400 \mathrm{~km}$. Or, we can use $\mu$ of Earth to be $398600 \mathrm{~km}^{3} / \mathrm{s}^{2}$ and period as $2 \pi a^{1.5} / \mathrm{J} \mu$

Note that 27.3 Earth days is then also the time Moon takes to complete one turn around its axis with respect to the stars, which is called a sidereal day of Moon. However, since Moon is bound to the Earth and Earth goes round the Sun in 365.25 days, the amount of time for Sun to return to the same position in the
sky for a Moon's observer is 29.5 days, which is called the synodic day. Half of this synodic day, of about 15 days is hence the sunlit or "daytime" on the Moon.

Note the similarity to Earth-Sun system where we know that Earth spin period is about 23 h and 56 m , whereas the day is reckoned as 24 hours to account for about 1 deg/day of Earth rotation around the Sun.

FAQ26: What makes lunar landing a big challenge that only 3 countries have done it so for?

The Moon's surface is quite undulated with large craters and rocks, and hence the landing site must be selected carefully. There is no GPS signal to navigate and so one has to depend on absolute inertial sensors and/or on real time observations of the terrain, and matched with maps stored as a reference.

Even as the horizontal component of velocity should be controlled to drive towards the target location (latitude \& longitude), the vertical velocity must be brought to near zero for safe and soft landing. The vertical component is subject to the acceleration due to gravity $\left(1.62 \mathrm{~m} / \mathrm{s}^{2}\right.$ near the surface, about $1 / 6^{\text {th }}$ of the value on Earth's surface). There is no atmosphere and so parachutecannot be used. One has to fire retro thrusters to bring the vertical velocity tozero.

FAQ27: What are the special challenges in landing in higher latitudes, particularly nearer to the South pole?

South polar region is ideal for a potential lunar outpost, due to the presence of frozen water and an abundance of resources. However, these regions have dark lighting conditions, besides huge craters. Typical temperatures of ranging from $50 \operatorname{deg} \mathrm{c}$ to $-230 \mathrm{deg} C$ is tough on the functioning of electronic instruments. The sun lighting exhibits seasonal patterns and so places constraints on the launch window. The Sun is always close to the horizon and hence some of the taller craters produce long shadows. These shadows and the poorer lighting conditions make it difficult to navigate the lander to the chosen site using prestored images of the region. For example, for landing near -70 deg latitude, the Sun is expected to be less than 20 deg above the horizon.

FAQ28: Now that the LOI was completed on 5 Aug, can we confirm that the LOI burn decreased the total energy corresponding to the hyperbolic arrival trajectory to that of the achieved elliptical orbit?

The LOI burn was for about 30 minutes, centered at around 7.30 pm IST. The total energy (TE) per unit mass in the arrival trajectory was around 0.373 units.

The positive sign here indicates it is not bound to Moon and would escape if left as it is. The post LOI orbit had a TE of around -. 224 units. The decrease of velocity (wrt Moon) from $2.41 \mathrm{~km} / \mathrm{s}$ to $2.168 \mathrm{~km} / \mathrm{s}$ assuming an impulsive burn around 7.30 pm IST would result in a change in KE of about 0.55 units. This removal of KE by the -deltaV burn accounts for the lowering of TE from 0.373 to -. 224 units.

FAQ29: Why Moon and Sun look discs of approximately same size from Earth? For celestial bodies, the large or small is basically the angle subtended at the eye. This angle is the ratio of the actual size to the distance to the object. The Sun is far away, but bigger in diameter so much compared to Moon. It so happens these two ratios are nearly equal (each about half a degree).

## STEM from Space

## Basic Physics \& Maths behind Chandrayaan-3 Mission- Set 5

FAQ30: With respect to Moon, what is the apparent movement of Earth, Sun and Stars?

From the Moon, Earth looks stationary, (i.e. in the same direction in the sky), provided you are the near side. On the far side of the Moon, Earth is never visible.

The Sun moves around with the synodic period, which is around 29.53 Earth days. Hence the sunlight movement on Moon's surface is about 12 deg/day. So it takes 2 hours to move through 1 deg.

With respect to the stars, the moon's period is about 27.321 Earth days, called its sidereal period. Hence, per day the stars have an apparent motion of 13.176 deg.

FAQ31: We were informed on the day of the launch itself, that the lander of Chandrayaan- 3 will touch down on Aug 23. How is the day and time of landing fixed?

The remarkable thing about this mission is the precision with which the events are planned and then making sure that each of them happen exactly as planned. The first thing to fix is the launch date and time. Based on the extensive mapping of the region of interest in landing, the landing site is selected with its latitude and longitude on the Moon's surface. Once this location is known, the landing date is decided by the lighting conditions, and continued daytime for the next 14 days.

From the Moon almanac, we know that for the chosen site of about 70 deg $S$ latitude and 32 deg longitude, the Sun rises on Aug 22. The required minimum Sun elevation of 6 deg will be achieved on Aug 23 which is the choice of the landing day. The precise time of landing then gets fixed from the landing orbit parameters that define the geometry with respect to the landing site.

FAQ32: On what considerations are the date and time of the Lunar Capture planned?

Working backwards from the landing day, we need about 2 weeks for the various lunar maneuvers and calibrations required for landing. Hence LOI should happen no later than 7 th Aug. The capture orbit should be parallel to the landing orbit, whose parameters have already chosen. Moon should be close to Earth's equator on the day of capture and CH 3 should have the rendezvous with

Moon near the equator. This fixes the day as $5^{\text {th }}$ Aug for LOI and the exact time will be arrived at from the orbit of Moon and the path of CH 3 on its lunar approach.

FAQ33: How is Lunar Transfer Trajectory planned and when does the LTI burn occur?

LVM-3 Launch-Range Safety has selected the CH-3 Inclination as 21.3 deg. Moon's orbit Inclination in this period is about 28 deg, with respect to the ecliptic plane. Near Ascending or Descending Node (AN or DN) the rendezvous is ideal. CH 3 is injected near perigee in the DN . Hence its argument of perigee is near 180 deg and the apsidal line nearly on the equatorial plane. Moon Crosses $A N$ on $7^{\text {th }}$ Aug. i.e. the declination with respect to equatorial plane is near zero. Moon capture around $5^{\text {th }}$ August is possible if the Ch3 orbit is stretched to have its apogee around and a little beyond Moon's distance, and it will be close to its ANon 5 Aug. (incidentally Moon is close to its perigee and its distance is around $363,300 \mathrm{~km}$; By the way do not miss to see the full Moon on 31 Aug when it will be at its biggest!). In such a highly elliptical orbit for Ch3 of apogee around 369600 km , the Moon encounter would happen close to, but prior to the apogee. The capture is aided by the relatively smaller velocity near apogee. One can compute the time of travel in this orbit from the perigee point (where the additional in-plane deltaV is imparted) to the rendezvous point. This is around
115.5 hours and subtracting this from the calculated capture time, we arrive at the firing for Lunar departure orbit as 31 Jul around 2.30 pm IST.

To compute the actual deltaV to be imparted, we can find the difference in perigee velocities of the orbit after the last EBN and the required orbit to reach Moon. Note that even after this addition, the new perigee velocity is less than the escape velocity at that altitude. It is so planned that CH 3 traverses this path and gets into the sphere of influence of Moon's gravity, it becomes a hyperbolic trajectory and unless immediately we do a retromanuver and lower the kinetic energy, we cannot get a lunar orbit.

FAQ34 : How was the launch day and time chosen for this mission?
To reach Moon Orbit distance, $\mathrm{CH}-3$ has to leave the LEO at $10.847 \mathrm{Km} / \mathrm{sec}$ speed, forming a the final TLI elliptical orbit. LVM3 can deliver the mass of 3900 kg of CH 3 at the perigee of 170 km , at the descending node at an inclination of 21.3 deg, with an apogee height of 36500 km . This amounts to an injection velocity about $10.27 \mathrm{~km} / \mathrm{s}$. The additional velocity to be imparted by the Satellite RCS system using the 440 N thruster requiring more than 90 minutes of firing. This cannot be imparted in one burn as that would cover too
big an arc around the perigee, making it inefficient. Hence it is planned in 5 parts through intermediate elliptical orbits, each with increasing orbit periods. Hence the Earth-centric phase would have to stretch over 2 weeks.

Hence we select a Launch day with 2 weeks before TLI. The RAAN of the orbit has to match with the expected Moon's RAAN on the day of rendezvous. This in turn, fixes the time of lift off on the chosen launch day.

FAQ35: Can you provide this interlinked timeline that brings out the logical sequence of the chosen parameters for this mission?

| Select landing | - Scientific interest <br> - Nature of terrain for safe and smooth landing <br> - Manueverability and reach of rover <br> - Acceptable terrain around the chosen one, to cater to dispersions |
| :---: | :---: |
| Landing date \& Time | - Moon's almanac <br> - Solar illumination at landing site <br> -Start of a 15 day daytime window |
| Date $\&$ time of Lunar capture | - Sufficient duration to shrink the orbit to 100 km circular <br> - Reach the landing orbit of $100 \times 30 \mathrm{~km}$ <br> -Lunarcraft health check <br> - Descent phase and final touch down |
| Departure | - CH3 should meet the Moon when Moon is around Earth Equator <br> - Velocity at perigee less than escape, but apogee just beyond Moon |
| Launch date \& | - Cantilre nehit plane chnilit he narallol fn I andino nrhit plane <br> - Range safety to decide inclination <br> - Sufficient time to raise apogee to the departure orbit with 5 burns <br> - Time of launch such that RANN to match Moon's RANN |

FAQ36: Can you show pictorially how the hyperbolic arrival trajectory was captured into lunar orbit, in terms of the distance from centre of Moon?

It is shown in the figure below. Note that the multiple revolutions around Moon after LOI are shown, whereas in the actual mission the orbit was lowered in its apolune distance, and the extreme values after the first and second LBN are below:

| Perilune (km) | Apolune (km) | $\begin{aligned} & \mathrm{rP} \\ & (\mathrm{~km}) \end{aligned}$ | rA (km) | SMA <br> (km) | vP $(\mathrm{km} / \mathrm{s})$ | vA $(\mathrm{km} / \mathrm{s})$ | Period (hrs) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 164 | 18074 | 1900 | 19810 | 10855 | 2.169445 | 0.208074 | 28.19838 |
| 170 | 4313 | 1906 | 6049 | 3977.5 | 1.977303 | 0.623035 | 6.25453 |
| 17 | 1437 | 1910 | 317 | 2541.5 | 1.789663 | 1.077295 | 3.194586 |



## Basic Physics \& Maths behind Chandrayaan-3 Mission- Set 6

A lot of students I am interacting with, raised a number of questions on the descent and landing phase of CH 3 mission. Many of them have solved the equations of motion in 2 dimensions, with a constant acceleration due to gravity ( $g$ ) acting vertically downward, and a constant velocity in the horizontal direction. They have solved and found "the trajectory of a projectile is a parabola". In the descent phase of the Moon we can again decompose into vertical and horizontal components and solve the equations of motion. But in this case, the accelerations cannot be assumed to be constant vertically due to considerable altitudes, and also there can be thruster-induced accelerations horizontally and/or vertically.

FAQ37: For quick reference can we have the velocities at perilune for the 100 km circular as well as $100 \times 30 \mathrm{~km}$ orbit around the Moon?

| Perilune <br> $(\mathrm{km})$ | Apolune <br> $(\mathrm{km})$ | rP <br> $(\mathrm{km})$ | rA <br> $(\mathrm{km})$ | SMA <br> $(\mathrm{km})$ | vP <br> $(\mathrm{km} / \mathrm{s})$ | vA <br> $(\mathrm{km} / \mathrm{s})$ | Period <br> $(\mathrm{hrs})$ |
| ---: | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 100 | 100 | 1836 | 1836 | 1836 | 1.63366 | 1.63366 | 1.9615 |
| 30 | 100 | 1766 | 1836 | 1801 | 1.68183 | 1.617708 | 1.90568 |

It is planned to de-boost from $100 \times 100 \mathrm{Km}$ orbit to $100 \times 30 \mathrm{Km}$ orbit.
FAQ38: What is the starting state and the required end state for landing?
At the start, the lander has a large horizontal velocity like $1681 \mathrm{~m} / \mathrm{s}$. This horizontal velocity is to be controlled by thrusting to let it move from the latitude \& longitude over which it is released (at a height of 30 km ) to the target latitude \& longitude of the chosen site. For safe and smooth landing, the lander should be vertically above the chosen target site, with zero horizontal velocity and near zero vertical velocity. Moon's gravity of $1.62 \mathrm{~m} / \mathrm{s}^{2}$ will make sure it moves down to the target, just like the apple came down on Newton's head !. The lander's legs are strong enough to withstand a small but finite speed at the time of touch down.

FAQ39: What are the major stages in the Moon-centric phase?
> Moon-bound maneuvers to reach $100 \times 100 \mathrm{~km}$ orbit
> Separation of Propellant module and Lander
> De-boost phase
> Rough Breaking
> Attitude of Lander held for checking and calibration
> Fine Breaking
> Vertical Landing
$\Rightarrow$ Normal operations for Lander and Rover

FAQ 40 : For navigation what sensors are used?
We are so much used to GPS navigation to guide us on the route from the starting point to our destination. Around the Moon this luxury is not available. In daylit condition of the sky the stars can be sensed, unlike from Earth where the atmosphere scatters and diffuses the sunlight. Even if they could be sensed, it cannot be used for closed loop guidance for this short span of time. For Lander's navigation, we use
> Inertial Navigation

- Gyros
- Accelerometers
> Absolute Navigation
- Light Detection and Ranging (LIDAR)
- High Resolution Camera (HRC)

The guidance algorithm is the software carried for the activating the controls for position and orientation.

FAQ41: What is the purpose of the "Absolute navigation" phase?
The absolute navigation phase occurs between the rough and fine breaking phases. The rough breaking phase would have ensured that the high horizontal velocity at the beginning would have come down to about 10\%, and also it would have picked some velocity component vertically down. Knowing the ratio of these two components, the guidance algorithm can orient the lander such that the two are simultaneously reduced. During this phase, the orientation will be maintained absolutely. This inertial attitude hold for a finite duration is used to finely calibrate the sensors and prepare the ground for handling the crucial fine breaking phase most efficiently.

FAQ42: What does the fine braking phase consist of ?
By comparing the prestored terrain images around the target site, the location of the target will be known. This information is used to bring the lander vertically over that spot, with zero horizontal velocity. After hovering for a short duration and further confirming the situation, the final touch down takes place safely and smoothly.

FAQ43: With whom can the Lander communicate?
The lander can communicate with the International Deep Space Network (ISDN); can communicate with Chandrayaan 2 Orbiter and also to the Rover.

FAQ44: Why was the first size reduction manoeuvre done only late night of Aug 6? Was it to give more time for orbit determination?

The orbit transfer from post LOI orbit to $170 \times 4313 \mathrm{~km}$ orbit, requires a retro firing at the perilune (reducing the speed from around $2.17 \mathrm{~km} / \mathrm{s}$ to 1.98 $\mathrm{km} / \mathrm{s}$ ). The period of LOI orbit is around 28.2 hours and so the next occurrence over perilune is only after one revolution. At this very first opportunity, the size was reduced. The time lapse between LOI and LBN\#1 was not based on time required for orbit determination.

FAQ45: After nearly 50 years, Russia has resumed its Moon exploration with Luna 25 launched on Aug 10, 2023, headed to land near the South pole. Will it "beat" India in the race?

Luna 25 is planned to land near Moon's south pole at the crater Boguslawsky. This location lies to the East of the planned landing location of CH 3 . Sun Light Movement on Moon Surface:12. 20 deg/day and Sun rises on Boguslawsky on 21 Aug, when it is still pre-dawn at our chosen site. This concept is quite similar to how on Earth different longitudes have different time (Singapore is 2.5 hours ahead of India, as it lies to our East). Our landing strategy requires a minimum of 6 deg Sun elevation and this is realized on 23 Aug, at the beginning of a 15day sunlit conditions.

The concept of a "race" between CH 3 and Luna-25 is entirely a media creation. In fact, in the words of our founder, Vikram Sarabhai, we never want to get into any such space race. There is renewal of interest in exploring Moon in this decade by a number of countries, as the launch costs have come down and the stage is set for private players and more commercial exploitation of space. CH 3 , Luna, Artemis etc. complement each other in getting to know the Moon better
and there is no competition. CH 3 and Luna 25 exploring different regions 118 km apart may, in fact, help the scientific community be better prepared for future missions.

FAQ46: What are the major payloads carried on CH 3 and why?
Firstly, note that all the payloads on CH 3 are indigenous. The Lander carries:
> RAMBHA-LP, which is to measure the surface plasma (i.e. ions and electrons)
> ChaSTE, which carries out thermo physical measurements
$>$ ILSA, To study the structure of lunar crust and mantle in terms of seismic activity

The Rover carries:
> APXS, an X-ray spectrometer to analyse the chemical composition as well as the availability of minerals on the lunar surface
> LIBS, Unlike APXS which uses alpha particles, LIBS uses a LASER beam to breakdown the lunar soil and estimate the composition of elements like $\mathrm{Mg}, \mathrm{Al}, \mathrm{Si}, \mathrm{K}, \mathrm{Ca}, \mathrm{Ti}$ and Fe .

In addition, the orbiting Propulsion module carries;
$>$ SHAPE, to study Earth in the near-infrared regions for spectropolarimetric signatures. (Yes, its is a selfie on Earth from Moon's distance!)

